# Automotive Aerodynamics & Body Engineering

## Unit III

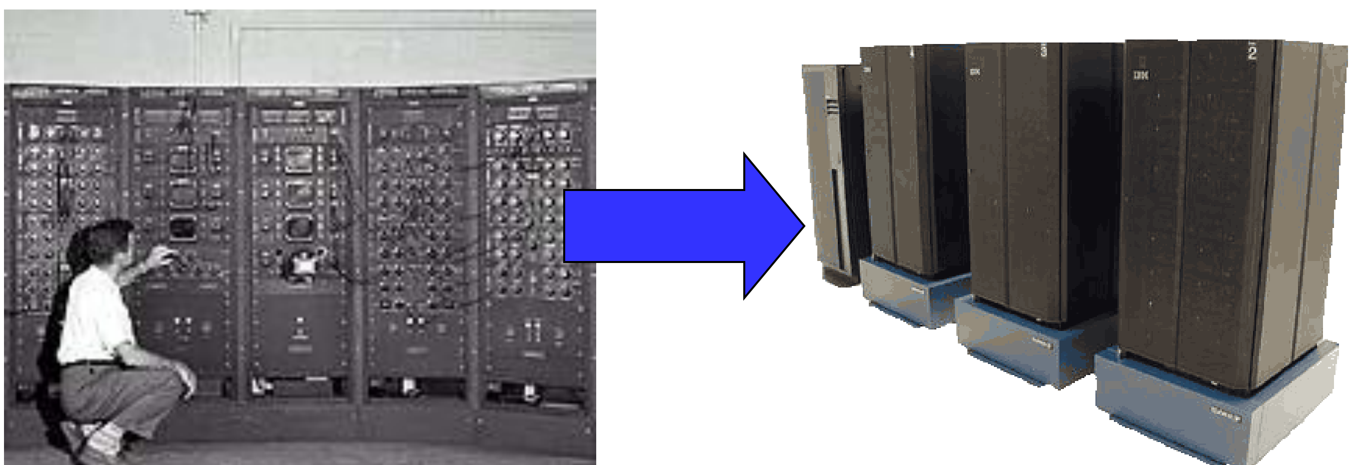
# **Computational Fluid Dynamics**

## Outline

- 1. What, why and where of CFD?
- 2. Modeling
- 3. Numerical methods
- 4. Types of CFD codes
- 5. CFD Educational Interface
- 6. CFD Process
- 7. Example of CFD Process
- 8. 58:160 CFD Labs

## What is CFD?

- CFD is the simulation of fluids engineering systems using modeling (mathematical physical problem formulation) and numerical methods (discretization methods, solvers, numerical parameters, and grid generations, etc.)
- Historically only Analytical Fluid Dynamics (AFD) and Experimental Fluid Dynamics (EFD).
- CFD made possible by the advent of digital computer and advancing with improvements of computer resources
- (500 flops, 1947→20 teraflops, 2003 →1.3 pentaflops, Roadrunner at Las Alamos National Lab, 2009.)

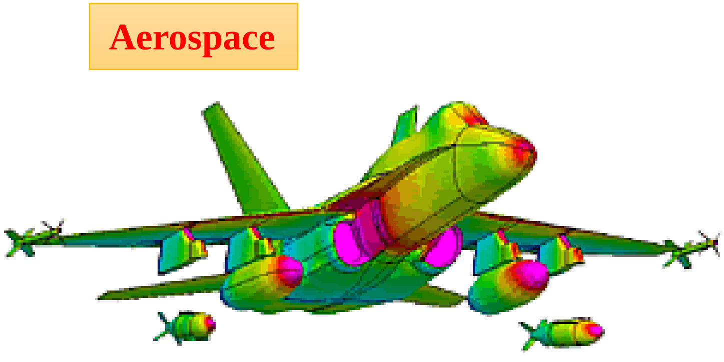


## Why use CFD?

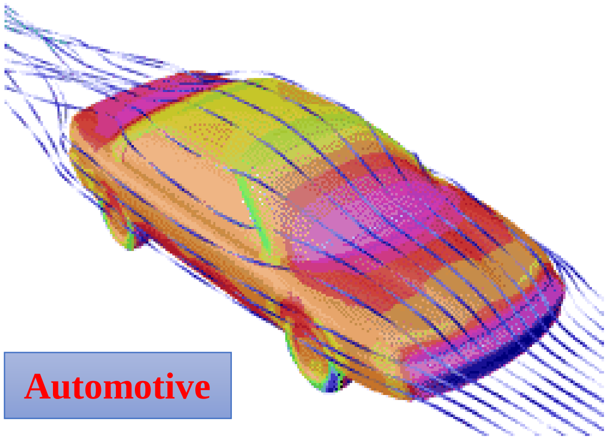
- Analysis and Design
  - 1. Simulation-based design instead of “build & test”
    - More cost effective and more rapid than EFD
    - CFD provides high-fidelity database for diagnosing flow field
  - 2. Simulation of physical fluid phenomena that are difficult for experiments
    - Full scale simulations (e.g., ships and airplanes)
    - Environmental effects (wind, weather, etc.)
    - Hazards (e.g., explosions, radiation, pollution)
    - Physics (e.g., planetary boundary layer, stellar evolution)
- Knowledge and exploration of flow physics

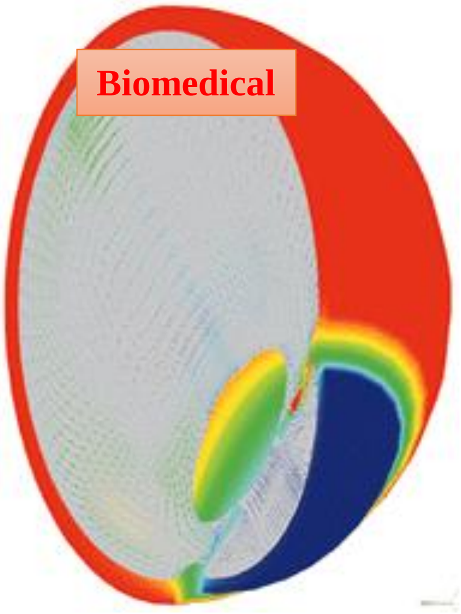
## Where is CFD used?

- ***Aerospace***
- ***Automotive***
- ***Biomedical***
- Chemical Processing
- HVAC
- Hydraulics
- Marine
- Oil & Gas
- Power Generation
- Sports



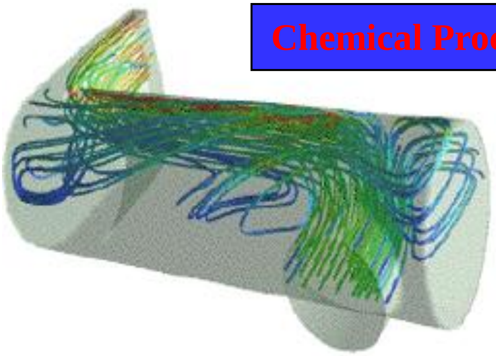

*F18 Store Separation*



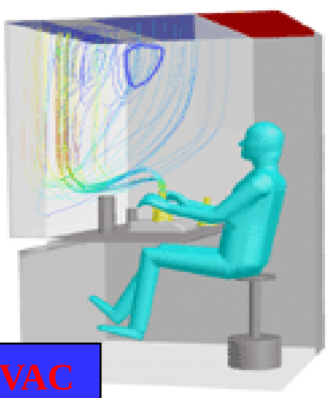




***Temperature and natural convection currents in the eye following laser heating.***

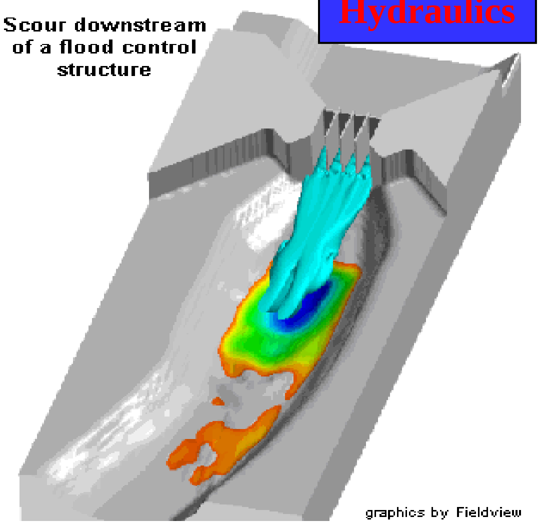
- Where is CFD used?
  - Aerospacee
  - Automotive
  - Biomedical
  - ***Chemical Processing***
  - ***HVAC***
  - ***Hydraulics***
  - Marine
  - Oil & Gas
  - Power Generation
  - Sports




***Polymerization reactor vessel - prediction of flow separation and residence time effects.***



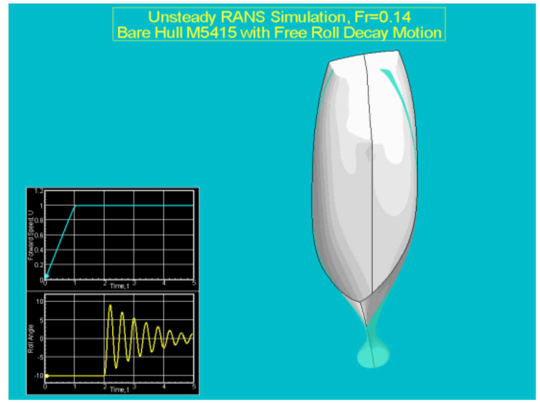

***Streamlines for workstation ventilation***



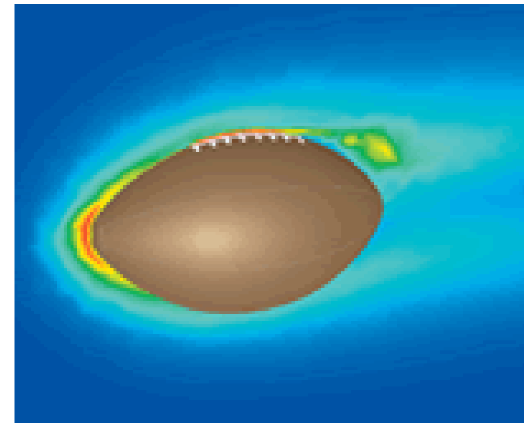
## Where is CFD used?

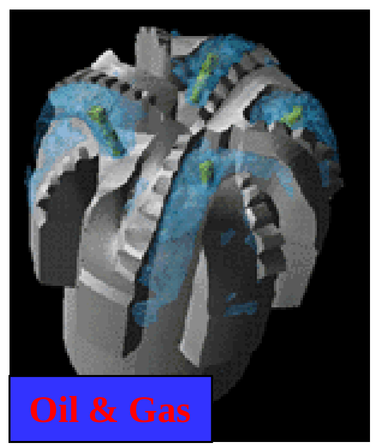
- Aerospace
- Automotive
- Biomedical
- Chemical Processing
- HVAC
- Hydraulics
- ***Marine***
- ***Oil & Gas***
- ***Power Generation***
- ***Sports***

**Marine (movie)**



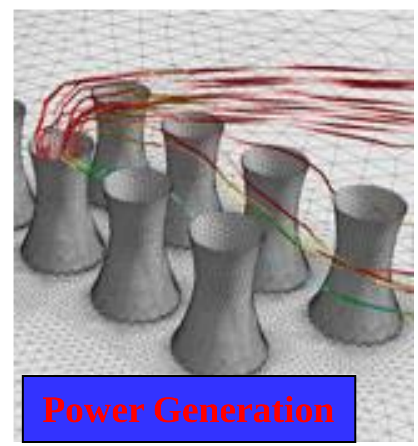

**Sports**





**Oil & Gas**

*Flow of lubricating mud over drill bit*



**Power Generation**

*Flow around cooling*

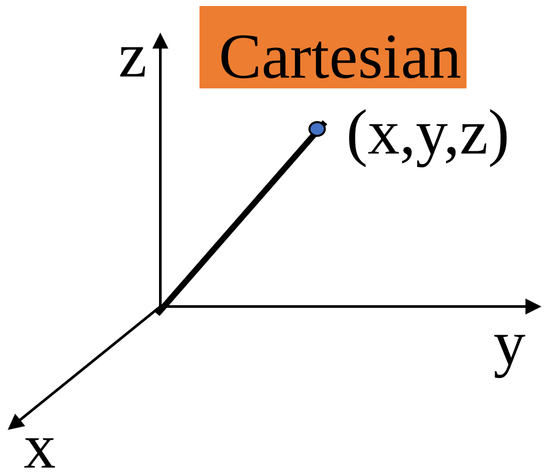
## Modeling

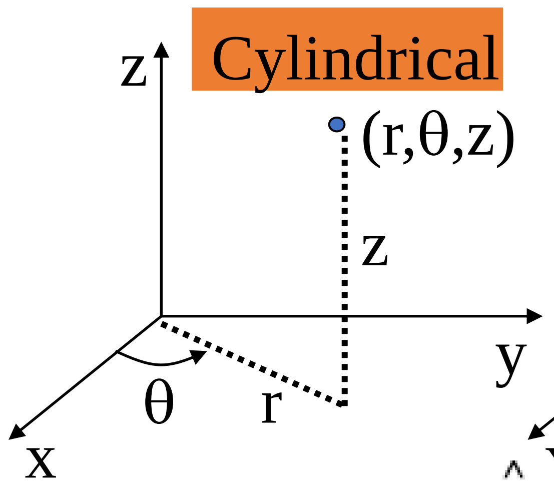
- Modeling is the mathematical physics problem formulation in terms of a continuous initial boundary value problem (IBVP)
- IBVP is in the form of Partial Differential Equations (PDEs) with appropriate boundary conditions and initial conditions.
- Modeling includes:
- 1. Geometry and domain
- 2. Coordinates
- 3. Governing equations
- 4. Flow conditions
- 5. Initial and boundary conditions
- 6. Selection of models for different applications

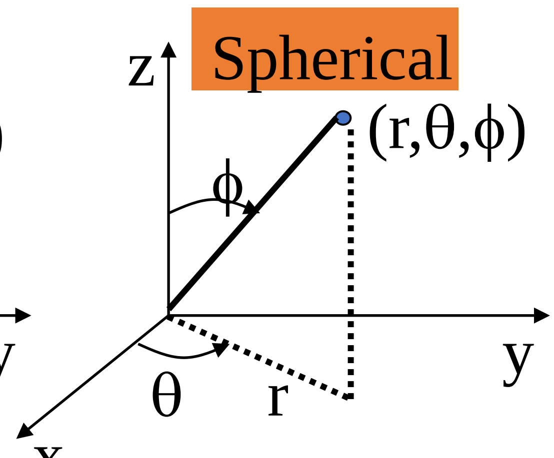
## Modeling (geometry and domain)

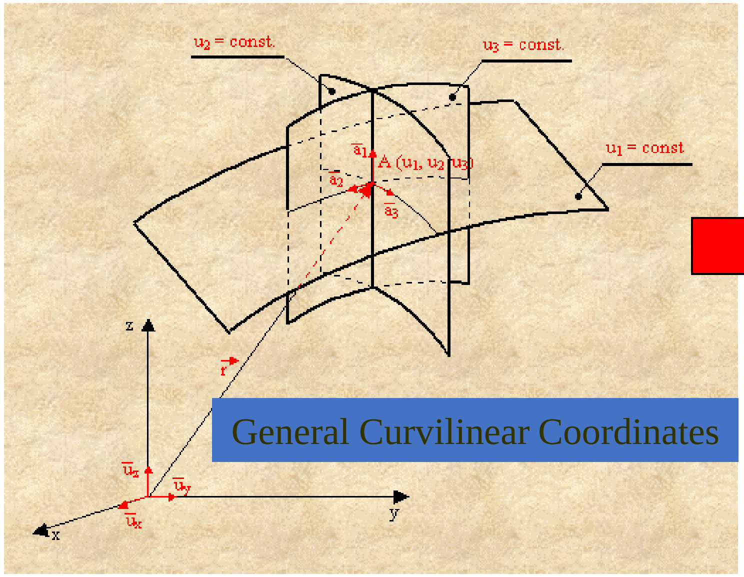
- Simple geometries can be easily created by few geometric parameters (e.g. circular pipe)
- Complex geometries must be created by the partial differential equations or importing the database of the geometry(e.g. airfoil) into commercial software
- Domain: size and shape
- Typical approaches
  - Geometry approximation
  - CAD/CAE integration: use of industry standards such as Parasolid, ACIS, STEP, or IGES, etc.
  - The three coordinates: Cartesian system (x,y,z), cylindrical system (r, θ, z), and spherical system(r, θ, Φ) should be appropriately chosen for a better resolution of the geometry (e.g. cylindrical for circular pipe).

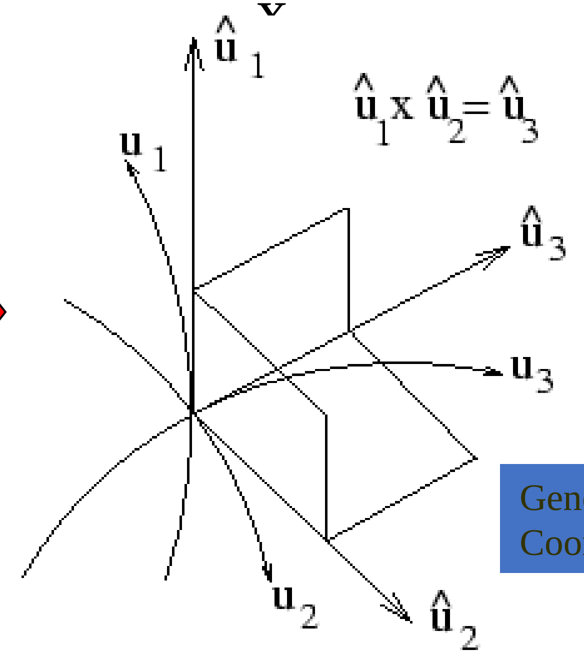
# Modeling (coordinates)




General orthogonal Coordinates

- Navier-Stokes equations (3D in Cartesian coordinates)

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial \hat{p}}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial \hat{p}}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]$$

$$\underbrace{\rho\frac{\partial w}{\partial t}}_{\text{Local acceleration}}+\underbrace{\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}}_{\text{Convection}}=\underbrace{-\frac{\partial \hat{p}}{\partial z}}_{\text{Piezometric pressure gradient}}+\underbrace{\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]}_{\text{Viscous terms}}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$ Continuity equation

$$p=\rho RT$$ Equation of state

$$R\frac{D^2R}{Dt^2}+\frac{3}{2}\left(\frac{DR}{Dt}\right)^2=\frac{p_v-p}{\rho_L}$$ Rayleigh Equation

## Modeling (flow conditions)

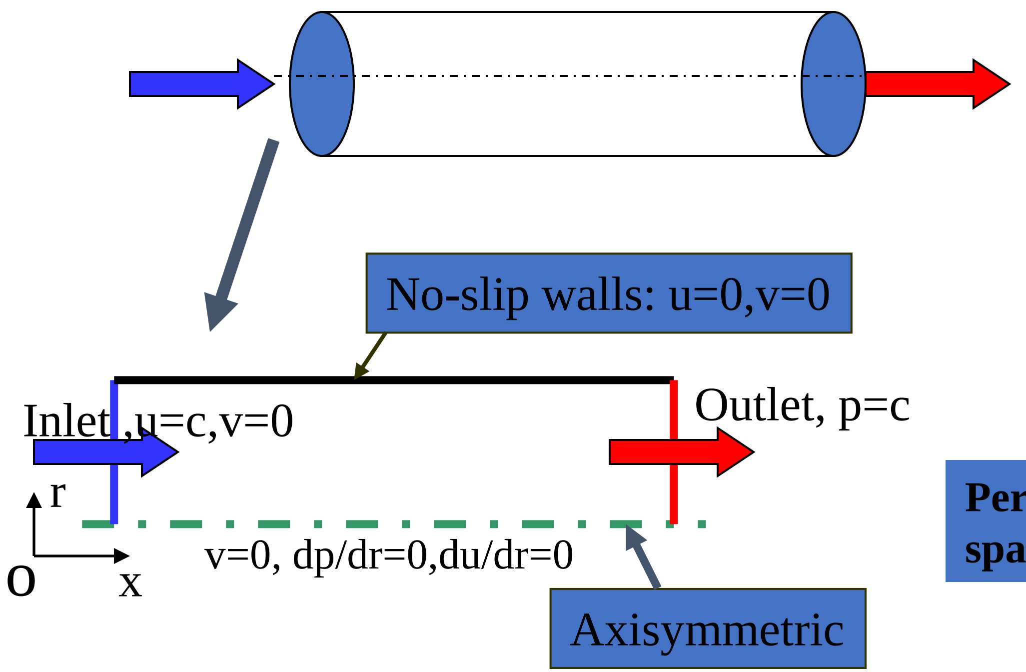
- Based on the physics of the fluids phenomena, CFD can be distinguished into different categories using different criteria
  - Viscous vs. inviscid (Re)
  - External flow or internal flow (wall bounded or not)
  - Turbulent vs. laminar (Re)
  - Incompressible vs. compressible (Ma)
  - Single- vs. multi-phase (Ca)
  - Thermal/density effects (Pr, g, Gr, Ec)
  - Free-surface flow (Fr) and surface tension (We)
  - Chemical reactions and combustion (Pe, Da) etc...

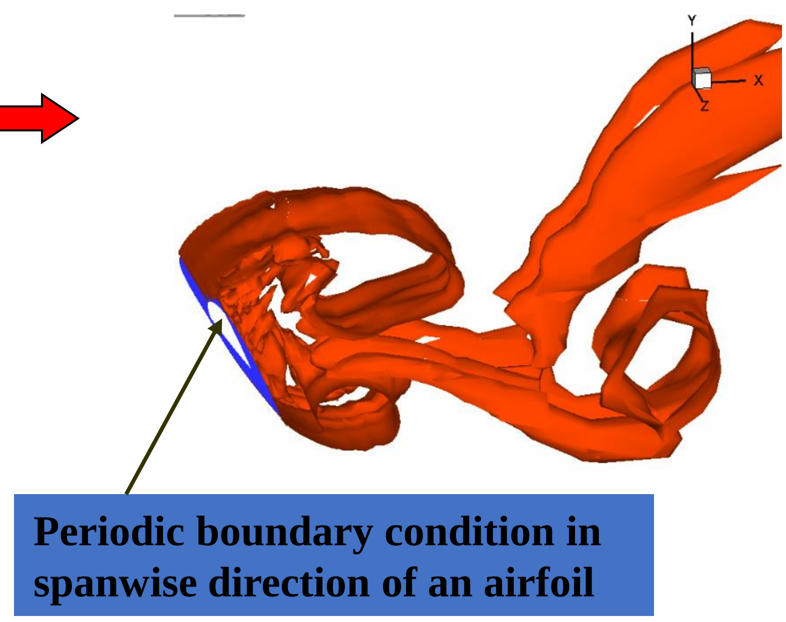
# Modeling (initial conditions)

- Initial conditions (ICS, steady/unsteady flows)
  - ICs should not affect final results and only affect convergence path, i.e. number of iterations (steady) or time steps (unsteady) need to reach converged solutions.
  - More reasonable guess can speed up the convergence
  - For complicated unsteady flow problems, CFD codes are usually run in the steady mode for a few iterations for getting a better initial conditions

# Modeling(boundary conditions)

- Boundary conditions: No-slip or slip-free on walls, periodic, inlet (velocity inlet, mass flow rate, constant pressure, etc.), outlet (constant pressure, velocity convective, numerical beach, zero-gradient), and non-reflecting (for compressible flows, such as acoustics), etc.



## Modeling (selection of models)

- CFD codes typically designed for solving certain fluid
- phenomenon by applying different models
  - Viscous vs. inviscid (Re)
  - Turbulent vs. laminar (Re, Turbulent models)
  - Incompressible vs. compressible (Ma, equation of state)
  - Single- vs. multi-phase (Ca, cavitation model, two-fluid
  - model)
  - Thermal/density effects and energy equation
  - (Pr, g, Gr, Ec, conservation of energy)
  - Free-surface flow (Fr, level-set & surface tracking model) and
  - surface tension (We, bubble dynamic model)
  - Chemical reactions and combustion (Chemical reaction
  - model) etc…

## Modeling (Turbulence and free surface models

- Turbulent flows at high Re usually involve both large and small scale
- vortical structures and very thin turbulent boundary layer (BL) near the wall

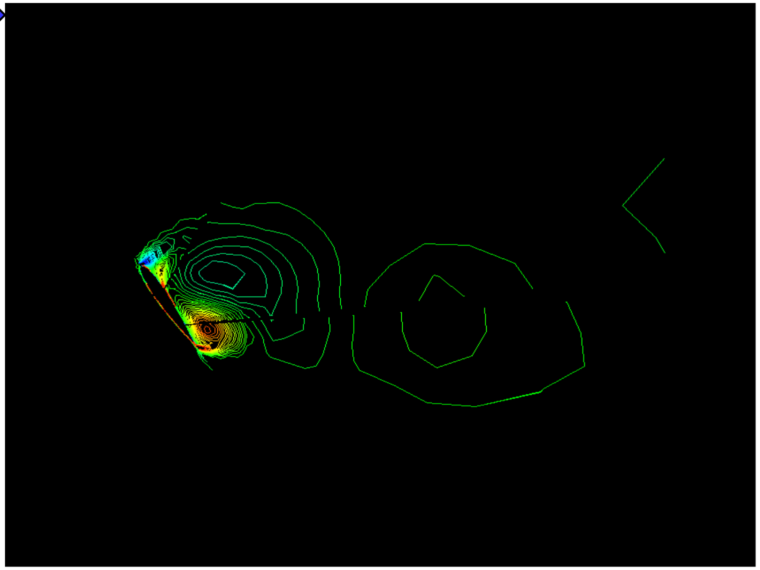
## Turbulent models:

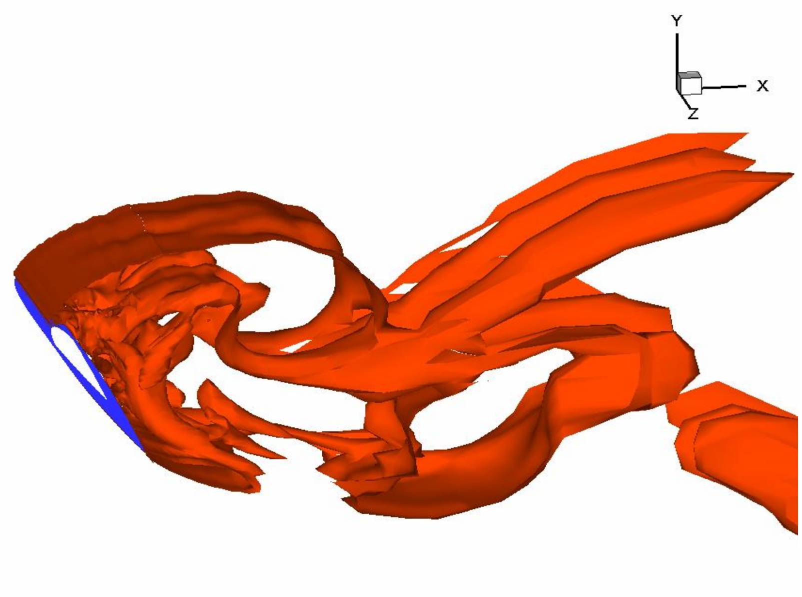
- DNS: most accurately solve NS equations, but too expensive
- for turbulent flows
- RANS: predict mean flow structures, efficient inside BL but excessive
- diffusion in the separated region.
- LES: accurate in separation region and unaffordable for resolving BL
- DES: RANS inside BL, LES in separated regions.

## Free-surface models:

- Surface-tracking method: mesh moving to capture free surface,
- limited to small and medium wave slopes
- Single/two phase level-set method: mesh fixed and level-set
- function used to capture the gas/liquid interface, capable of
- studying steep or breaking waves.

**URANS**, Re=$10^5$, contour of vorticity for turbulent flow around NACA12 with angle of attack 60 degrees






**DES**, Re=$10^5$, Iso-surface of Q criterion (0.4) for turbulent flow around NACA12 with angle of attack 60 degrees

**URANS**, Wigley Hull pitching and heaving

## Numerical methods

- The continuous Initial Boundary Value Problems (IBVPs) are discretized into algebraic equations using numerical methods. Assemble the system of algebraic equations and solve the system to get approximate solutions
- Numerical methods include:
  - 1. Discretization methods
  - 2. Solvers and numerical parameters
  - 3. Grid generation and transformation
  - 4. High Performance Computation (HPC) and post-
  - processing

## Discretization methods

- Finite difference methods (straightforward to apply, usually for regular grid) and finite volumes and finite element methods (usually for irregular meshes)
- Each type of methods above yields the same solution if the grid is fine enough. However, some methods are more suitable to some cases than others
- Finite difference methods for spatial derivatives with different order of accuracies can be derived using Taylor expansions, such as 2$^{nd}$ order upwind scheme, central differences schemes, etc.
- Higher order numerical methods usually predict higher order of accuracy for CFD, but more likely unstable due to less numerical dissipation
- Temporal derivatives can be integrated either by the explicit method (Euler, Runge-Kutta, etc.) or implicit method (e.g. Beam-Warming method)

## Discretization methods (Cont'd)

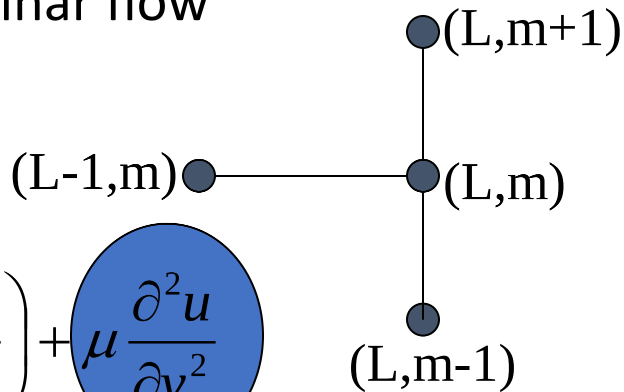
- Explicit methods can be easily applied but yield conditionally stable Finite Different Equations (FDEs), which are restricted by the time step; Implicit methods are unconditionally stable, but need efforts on efficiency.
- Usually, higher-order temporal discretization is used when the spatial discretization is also of higher order.
- Stability: A discretization method is said to be stable if it does not magnify the errors that appear in the course of numerical solution process.
- Pre-conditioning method is used when the matrix of the linear algebraic system is ill-posed, such as multi-phase flows, flows with a broad range of Mach numbers, etc.
- Selection of discretization methods should consider efficiency, accuracy and special requirements, such as shock wave tracking.

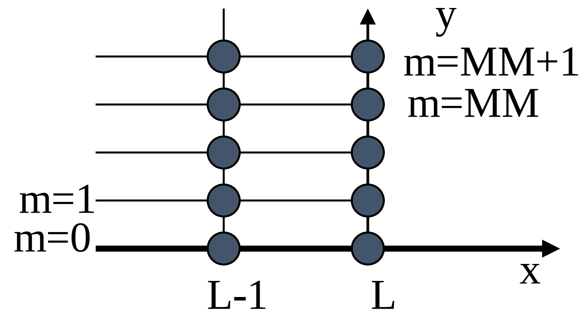
# Discretization methods (example)

- 2D incompressible laminar flow boundary layer

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial}{\partial x}\left(\frac{p}{e}\right)+\mu\frac{\partial^2 u}{\partial y^2}$$




$$u\frac{\partial u}{\partial x}=\frac{u_m^l}{\Delta x}\left[u_m^l-u_m^{l-1}\right]$$

$$\mu\frac{\partial^2 u}{\partial y^2}=\frac{\mu}{\Delta y^2}\left[u_{m+1}^l-2u_m^l+u_{m-1}^l\right]$$

$$v\frac{\partial u}{\partial y}=\frac{v_m^l}{\Delta y}\left[u_{m+1}^l-u_m^l\right]\quad \text{FD}\quad \text{Sign}(v_m^l)<0$$

$$=\frac{v_m^l}{\Delta y}\left[u_m^l-u_{m-1}^l\right]\quad \text{BD}\quad \text{Sign}(v_m^l)>0$$

2nd order central difference i.e., theoretical order of accuracy $P_{kest}=2$.

1st order upwind scheme, i.e., theoretical order of accuracy $P_{kest}=1$

$$\overset{B_2}{\boxed{\left[\frac{u_m^l}{\Delta x}+v_m^l\,\frac{-\frac{1}{\Delta y}FD}{\frac{1}{\Delta y}BD}-\frac{2\mu}{\Delta y^2}\right]}}u_m^l+\overset{B_3}{\boxed{\left[\frac{\mu}{\Delta y^2}+\frac{v_m^l}{\Delta y}FD\right]}}u_{m+1}^l+\overset{B_1}{\boxed{\left[\frac{\mu}{\Delta y^2}-\frac{v_m^l}{\Delta y}BD\right]}}u_{m-1}^l$$

$$=\underset{B_4}{\boxed{\frac{u_m^l}{\Delta x}}}u_m^{l-1}-\frac{\partial}{\partial x}(p/e)_m^l$$

$$B_1u_{m-1}^l+B_2u_m^l+B_3u_{m+1}^l=B_4u_m^{l-1}-\frac{\partial}{\partial x}\left(p/e\right)_m^l$$

$$\begin{bmatrix} B_2 & B_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_1 & B_2 & B_3 & 0 & 0 & 0 & 0 & 0 \\ & & \bullet & \bullet & \bullet & \bullet & & \\ 0 & 0 & 0 & 0 & 0 & B_1 & B_2 & B_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & B_1 & B_2 \end{bmatrix}\times\begin{bmatrix} u_1^l \\ \bullet \\ \bullet \\ \bullet \\ u_{mm}^l \end{bmatrix}=\begin{bmatrix} B_4u_1^{l-1}-\frac{\partial}{\partial x}\left(\frac{p}{e}\right)_1^l \\ \bullet \\ \bullet \\ \bullet \\ B_4u_{mm}^{l-1}-\frac{\partial}{\partial x}\left(\frac{p}{e}\right)_{mm}^l \end{bmatrix}$$

**Solve it using Thomas algorithm**

**To be stable, Matrix has to be Diagonally dominant.**

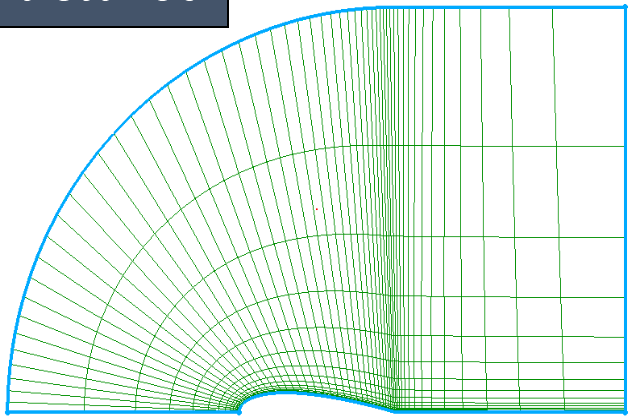
## Solvers and numerical parameters

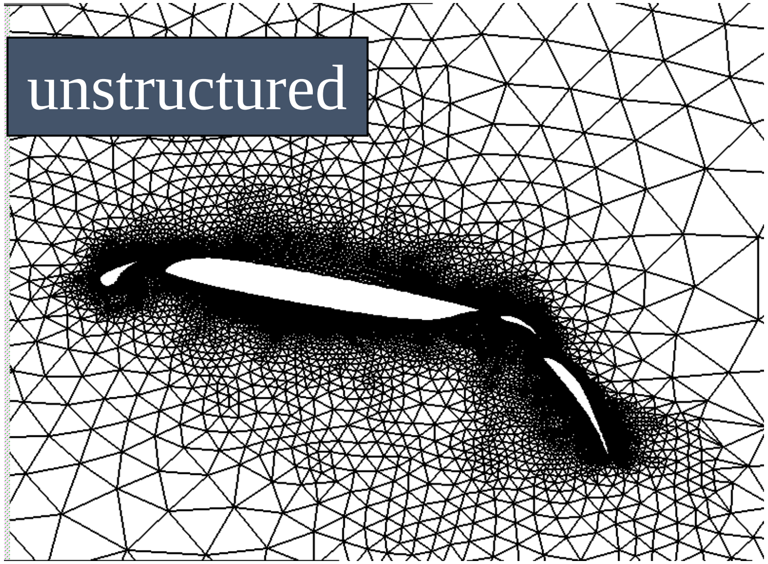
- **Solvers** include: tridiagonal, pentadiagonal solvers, PETSC solver, solution-adaptive solver, multi-grid solvers, etc.
- **Solvers** can be either direct (Cramer's rule, Gauss elimination, LU decomposition) or iterative (Jacobi method, Gauss-Seidel method, SOR method)
- **Numerical parameters** need to be specified to control the calculation.
  - Under relaxation factor, convergence limit, etc.
  - Different numerical schemes
  - Monitor residuals (change of results between iterations)
  - Number of iterations for steady flow or number of time steps for unsteady flow
  - Single/double precisions

## Numerical methods (grid generation)

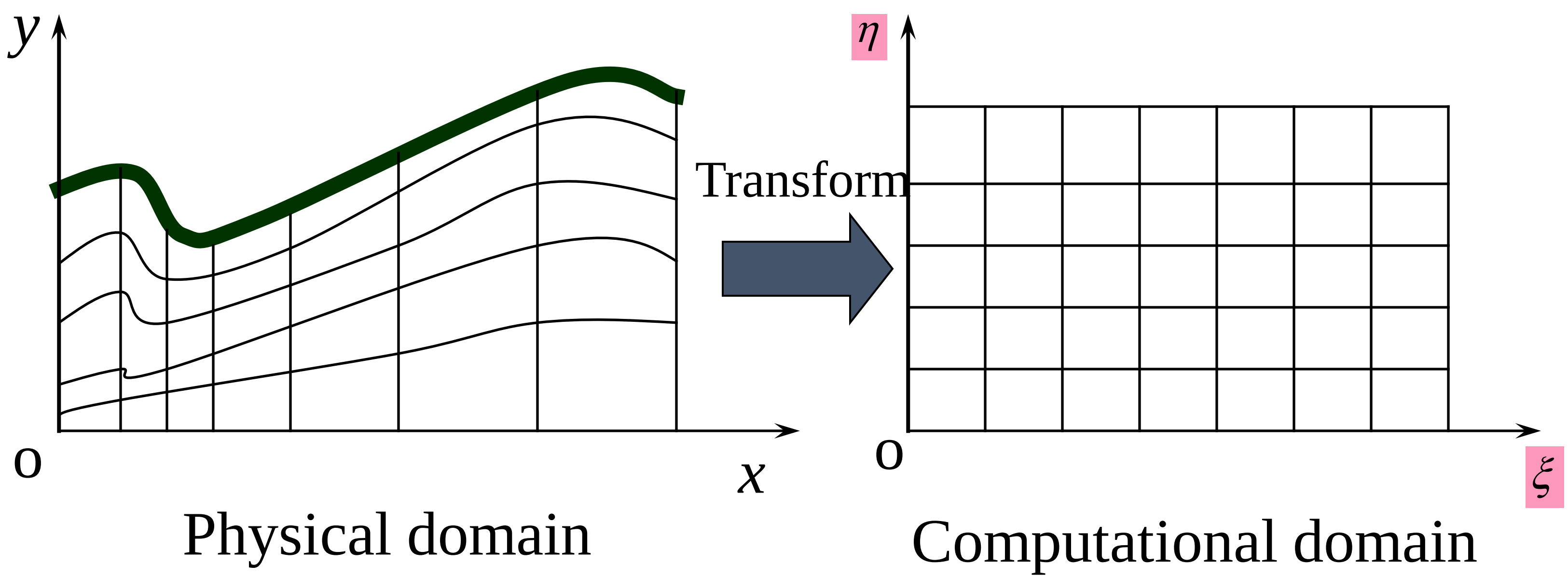
- Grids can either be structured (hexahedral) or unstructured (tetrahedral). Depends upon type of discretization scheme and application
  - Scheme
    - Finite differences: structured
    - Finite volume or finite element: structured or unstructured
  - Application
    - Thin boundary layers best resolved with highly-stretched structured grids
    - Unstructured grids useful for complex geometries
    - Unstructured grids permit automatic adaptive refinement based on the pressure gradient, or regions interested (FLUENT)

structured





# Numerical methods (grid transformation)




- Transformation between physical (x,y,z) and computational (ξ,η,ζ) domains, important for body-fitted grids. The partial derivatives at these two domains have the relationship (2D as an example)

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial x} = \xi_x \frac{\partial f}{\partial \xi} + \eta_x \frac{\partial f}{\partial \eta}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y} = \xi_y \frac{\partial f}{\partial \xi} + \eta_y \frac{\partial f}{\partial \eta}$$

## High performance computing

- CFD computations (e.g. 3D unsteady flows) are usually very expensive which requires parallel high performance supercomputers (e.g. IBM 690) with the use of multi-block technique.
- As required by the multi-block technique, CFD codes need to be developed using the Massage Passing Interface (MPI) Standard to transfer data between different blocks.
- Emphasis on improving:
  - Strong scalability, main bottleneck pressure Poisson solver for incompressible flow.
  - Weak scalability, limited by the memory requirements.

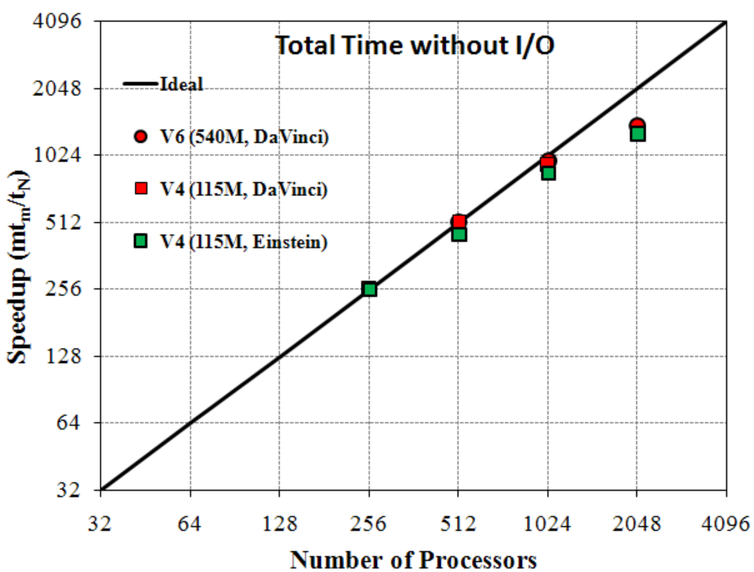



Figure: Strong scalability of total times without I/O for CFDShip-Iowa V6 and V4 on NAVO Cray XT5 (Einstein) and IBM P6 (DaVinci) are compared with ideal scaling.

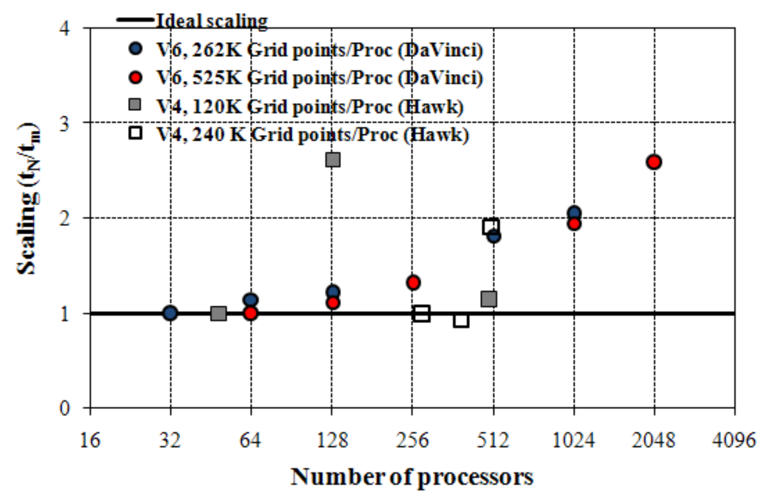
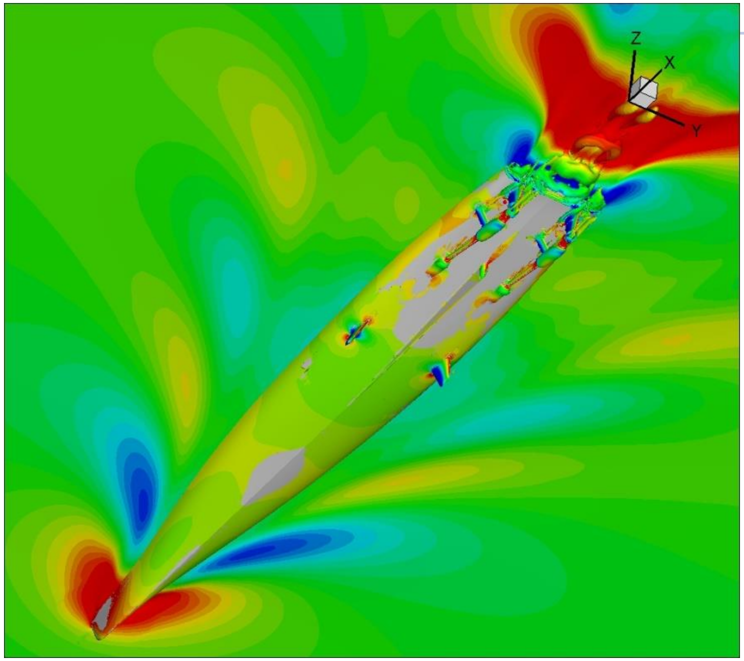



Figure: Weak scalability of total times without I/O for CFDShip-Iowa V6 and V4 on IBM P6 (DaVinci) and SGI Altix (Hawk) are compared with ideal scaling.

## Post-Processing

Post-processing: 1. Visualize the CFD results (contour, velocity vectors, streamlines, pathlines, streak lines, and iso-surface in 3D, etc.), and 2. CFD UA: verification and validation using EFD data (more details later)

Post-processing usually through using commercial software



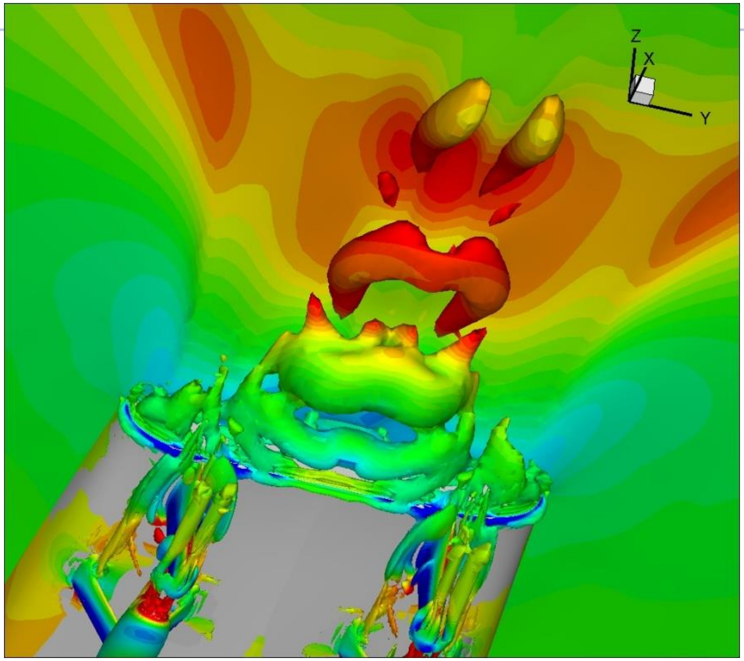




Figure: Isosurface of Q=300 colored using piezometric pressure, free=surface colored using z for fully appended Athena, Fr=0.25, Re=$2.9\times10^{8}$. Tecplot360 is used for visualization.

# Types of CFD codes

- Commercial CFD code: FLUENT, Star-CD, CFDRC, CFX/AEA, etc.
- Research CFD code: CFDSHIP-IOWA
- Public domain software (PHI3D, HYDRO, and WinpipeD, etc.)
- Other CFD software includes the Grid generation software (e.g. Gridgen, Gambit) and flow visualization software (e.g. Tecplot, FieldView)

FLUENT

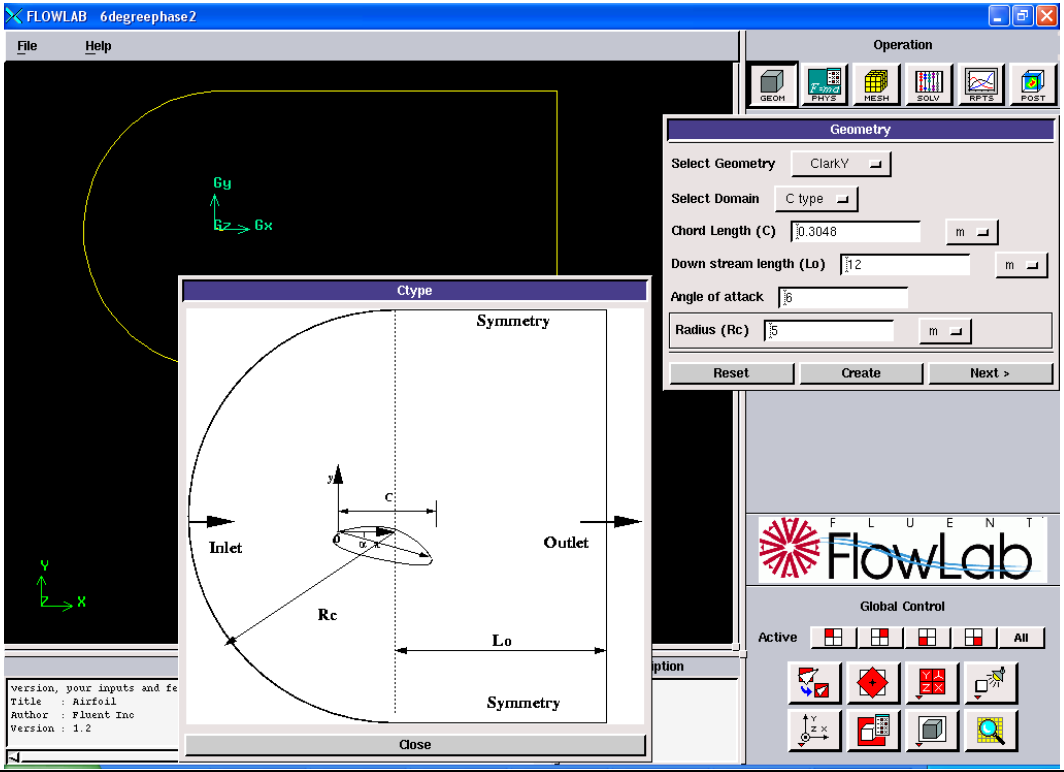
STAR CD

CD adapco Group

Gridgen

Reliable CFD Meshing


# CFD Educational Interface




| Lab1: Pipe Flow | Lab 2: Airfoil Flow | Lab3: Diffuser | Lab4: Ahmed car |
|---|---|---|---|
| 1. Definition of “CFD Process”<br>2. Boundary conditions<br>3. Iterative error<br>4. Grid error<br>5. Developing length of laminar and turbulent pipe flows.<br>6. Verification using AFD<br>7. Validation using EFD | 1. Boundary conditions<br>2. Effect of order of accuracy on verification results<br>3. Effect of grid generation topology, “C” and “O” Meshes<br>4. Effect of angle of attack/turbulent models on flow field<br>5. Verification and Validation using EFD | 1. Meshing and iterative convergence<br>2. Boundary layer separation<br>3. Axial velocity profile<br>4. Streamlines<br>5. Effect of turbulence models<br>6. Effect of expansion angle and comparison with LES, EFD, and RANS. | 1. Meshing and iterative convergence<br>2. Boundary layer separation<br>3. Axial velocity profile<br>4. Streamlines<br>5. Effect of slant angle and comparison with LES, EFD, and RANS. |

## CFD process

- Purposes of CFD codes will be different for different applications: investigation of bubble-fluid interactions for bubbly flows, study of wave induced massively separated flows for free-surface, etc.
- Depend on the specific purpose and flow conditions of the problem, different CFD codes can be chosen for different applications (aerospace, marines, combustion, multi-phase flows, etc.)
- Once purposes and CFD codes chosen, “CFD process” is the steps to set up the IBVP problem and run the code:
- 1. Geometry
- 2. Physics
- 3. Mesh
- 4. Solve
- 5. Reports
- 6. Post processing

| Geometry | Physics | Mesh | Solve | Reports | Post-Processing |
|---|---|---|---|---|---|
| Select Geometry | Heat Transfer ON/OFF | Unstructured (automatic/ manual) | Steady/ Unsteady | Forces Report (lift/drag, shear stress, etc) | Contours |
| Geometry Parameters | Compressible ON/OFF | Structured (automatic/ manual) | Iterations/ Steps | XY Plot | Vectors |
| Domain Shape and Size | Flow properties | | Convergent Limit | Verification | Streamlines |
| | Viscous Model | | Precisions (single/ double) | Validation | |
| | Boundary Conditions | | Numerical Scheme | | |
| | Initial Conditions | | | | |

# CFD Process

## Geometry

- Selection of an appropriate coordinate
- Determine the domain size and shape
- Any simplifications needed?
- What kinds of shapes needed to be used to best resolve the geometry? (lines, circular, ovals, etc.)
- For commercial code, geometry is usually created using commercial software (either separated from the commercial code itself, like Gambit, or combined together, like FlowLab)
- For research code, commercial software (e.g. Gridgen) is used.

Physics

- Flow conditions and fluid properties
- 1. **Flow conditions**: inviscid, viscous, laminar, or turbulent, etc.
- 2. **Fluid properties**: density, viscosity, and
- thermal conductivity, etc.
  - 3. Flow conditions and properties usually presented in dimensional form in industrial commercial CFD software, whereas in non-dimensional variables for research codes.
- Selection of models: different models usually fixed by codes, options for user to choose
- Initial and Boundary Conditions: not fixed by codes, user needs specify them for different applications.

**Mesh**

- Meshes should be well designed to resolve important flow features which are dependent upon flow condition parameters (e.g., Re), such as the grid refinement inside the wall boundary layer
- Mesh can be generated by either commercial codes (Gridgen, Gambit, etc.) or research code (using algebraic vs. PDE based, conformal mapping, etc.)
- The mesh, together with the boundary conditions need to be exported from commercial software in a certain format that can be recognized by the research CFD code or other commercial CFD software.

# Solve

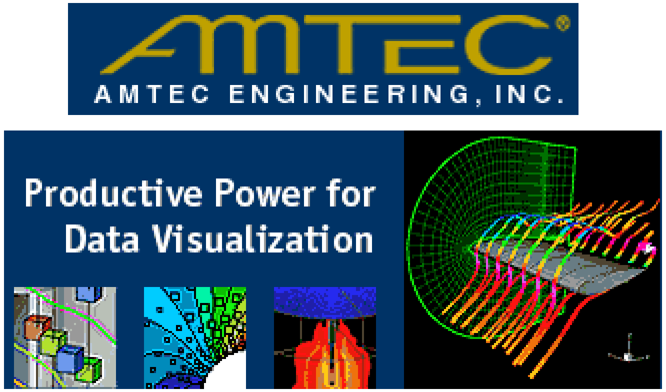
- Setup appropriate numerical parameters
- Choose appropriate Solvers
- Solution procedure (e.g. incompressible flows)
- Solve the momentum, pressure Poisson equations and get flow field quantities, such as velocity, turbulence intensity, pressure and integral quantities (lift, drag forces)

## Reports

- Reports saved the time history of the residuals of the velocity, pressure and temperature, etc.
- Report the integral quantities, such as total pressure drop, friction factor (pipe flow), lift and drag coefficients (airfoil flow), etc.
- XY plots could present the centerline velocity/pressure distribution, friction factor distribution (pipe flow), pressure coefficient distribution (airfoil flow).
- AFD or EFD data can be imported and put on top of the XY plots for validation

## Post-processing

- Analysis and visualization
  - Calculation of derived variables
    - Vorticity
    - Wall shear stress
  - Calculation of integral parameters: forces, moments
  - Visualization (usually with commercial software)
    - Simple 2D contours
    - 3D contour isosurface plots
    - Vector plots and streamlines (streamlines are the lines whose tangent direction is the same as the velocity vectors)
    - Animations



# Post-processing (Uncertainty Assessment)

- **Simulation error**: the difference between a simulation result S and the truth T (objective reality), assumed composed of additive modeling $\delta_{SM}$ and numerical $\delta_{SN}$ errors:

Error: $\delta_S = S - T = \delta_{SM} + \delta_{SN}$ Uncertainty: $U_S^2 = U_{SM}^2 + U_{SN}^2$

- **Verification**: process for assessing simulation numerical uncertainties $U_{SN}$ and, when conditions permit, estimating the sign and magnitude Delta $\delta^*_{SN}$ of the simulation numerical error itself and the uncertainties in that error estimate $U_{SN}$

$$\delta_{SN} = \delta_I + \delta_G + \delta_T + \delta_P = \delta_I + \sum_{j=1}^{J} \delta_j \qquad U_{SN}^2 = U_I^2 + U_G^2 + U_T^2 + U_P^2$$

*I: Iterative, G : Grid, T: Time step, P: Input parameters*

- **Validation**: process for assessing simulation modeling uncertainty $U_{SM}$ by using benchmark experimental data and, when conditions permit, estimating the sign and magnitude of the modeling error $\delta_{SM}$ itself.

$$E = D - S = \delta_D - (\delta_{SM} + \delta_{SN}) \qquad U_V^2 = U_D^2 + U_{SN}^2$$

*D*: EFD Data; $U_V$: Validation Uncertainty

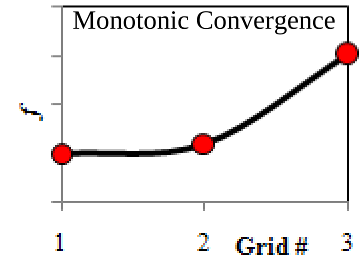
$|E| < U_V$ Validation achieved

# Post-processing (UA, Verification)

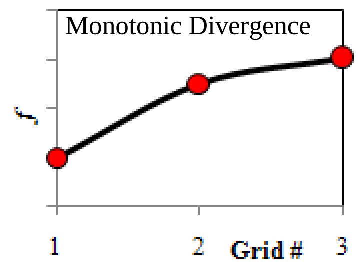
- Convergence studies: Convergence studies require a minimum of m=3 solutions to evaluate convergence with respective to input parameters. Consider the solutions corresponding to fine $\hat{S}_{k1}$, medium $\hat{S}_{k2}$, and coarse meshes $\hat{S}_{k3}$

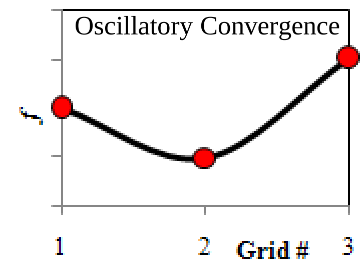
$$\varepsilon_{k21} = \hat{S}_{k2} - \hat{S}_{k1} \qquad \varepsilon_{k32} = \hat{S}_{k3} - \hat{S}_{k2}$$

$$R_k = \varepsilon_{k21} / \varepsilon_{k32}$$




(i). Monotonic convergence: $0<R_k<1$
(ii). Oscillatory Convergence: $R_k<0$; $|R_k|<1$
(iii). Monotonic divergence: $R_k>1$
(iv). Oscillatory divergence: $R_k<0$; $|R_k|>1$




- Grid refinement ratio: uniform ratio of grid spacing between meshes.

$$r_k = \Delta x_{k_2} / \Delta x_{k_1} = \Delta x_{k_3} / \Delta x_{k_2} = \Delta x_{k_m} / \Delta x_{k_{m-1}}$$

| | |
|---|---|
| Post-processing (Verification, RE) | Generalized Richardson Extrapolation (RE): For **monotonic convergence**, generalized RE is used to estimate the error $\delta^*_k$ and order of accuracy $p_k$ due to the selection of the *kth* input parameter. |
| | The error is expanded in a power series expansion with integer powers of $\Delta x_k$ as a finite sum. |
| | The accuracy of the estimates depends on how many terms are retained in the expansion, the magnitude (importance) of the higher-order terms, and the validity of the assumptions made in RE theory |

# Post-processing (Verification, RE)

$$\delta_{SN} = \delta_{SN}^{*} + \varepsilon_{SN}$$ $\varepsilon_{SN}$ is the error in the estimate

$$S_C = S - \delta_{SN}^{*}$$ $S_C$ is the numerical benchmark

Power series expansion

$$\hat{S}_{k_m} = S_{k_m} - \delta_{I_{k_m}}^{*} = S_C + \delta_{k_m}^{*} + \sum_{j=1,\, j\neq k}^{J} \delta_{jm}^{*}$$

Finite sum for the *k*th parameter and *m*th solution

$p_k$

$$\hat{S}_{k_m} = S_C + \sum_{i=1}^{n} \left(\Delta x_{k_m}\right)^{p_k^{(i)}} g_k^{(i)} + \sum_{j=1,\, j\neq k}^{J} \delta_{jm}^{*}$$

order of accuracy for the *i*th term

$$\hat{S}_{k_2} = S_C + \left(r_k \Delta x_{k_1}\right)^{p_k^{(1)}} g_k^{(1)} + \sum_{j=1,\, j\neq k}^{J} \delta_{j2}^{*}$$

$$\hat{S}_{k_3} = S_C + \left(r_k^2 \Delta x_{k_1}\right)^{p_k^{(1)}} g_k^{(1)} + \sum_{j=1,\, j\neq k}^{J} \delta_{j3}^{*}$$

Three equations with three unknowns

$$p_k = \frac{\ln\left(\varepsilon_{k_{32}} / \varepsilon_{k_{21}}\right)}{\ln\left(r_k\right)} \qquad \delta_{k_1}^{*} = \delta_{RE_{k1}}^{*} = \frac{\varepsilon_{k_{21}}}{r_k^{p_k} - 1}$$

- Monotonic Convergence: Generalized Richardson Extrapolation

**1. Correction factors**

$$p_k = \frac{\ln\left(\varepsilon_{k32}/\varepsilon_{k21}\right)}{\ln\left(r_k\right)} \rightarrow C_k = \frac{r_k^{p_k} - 1}{r_k^{p_{k_{est}}} - 1}$$

$$\delta^*_{RE_{k1}} = \frac{\varepsilon_{k21}}{r_k^{p_k} - 1}$$

$$U_k = \begin{cases} \left[9.6\left(1-C_k\right)^2 + 1.1\right]\left|\delta^*_{RE_{k1}}\right| & |1-C_k| < 0.125 \\ \left[2\left|1-C_k\right| + 1\right]\left|\delta^*_{RE_{k1}}\right| & |1-C_k| \ge 0.125 \end{cases}$$

$$U_{kc} = \begin{cases} \left[2.4(1-C_k)^2 + 0.1\right]\left|\delta^*_{RE_{k1}}\right| & |1-C_k| < 0.25 \\ \left[\left|1-C_k\right|\right]\left|\delta^*_{RE_{k1}}\right| & |1-C_k| \ge 0.25 \end{cases}$$

$p_{k_{est}}$ is the theoretical order of accuracy, 2 for 2nd order and 1 for 1st order schemes

$C_k$ is the correction factor

$U_k$ is the uncertainties based on fine mesh solution. $U_{kc}$ is the uncertainties based on numerical benchmark $S_C$

**2. GCI approach** $U_k = F_s\left|\delta^*_{RE_{k1}}\right|$ $U_{kc} = \left(F_s - 1\right)\left|\delta^*_{RE_{k1}}\right|$ $F_S$: Factor of Safety

- Oscillatory Convergence: Uncertainties can be estimated, but without signs and magnitudes of the errors. $U_k = \frac{1}{2}\left(S_U - S_L\right)$
- Divergence
- In this course, only grid uncertainties studied. So, all the variables with subscribe symbol k will be replaced by g, such as "$U_k$" will be "$U_g$"

## Post-processing (Verification, Asymptotic Range)

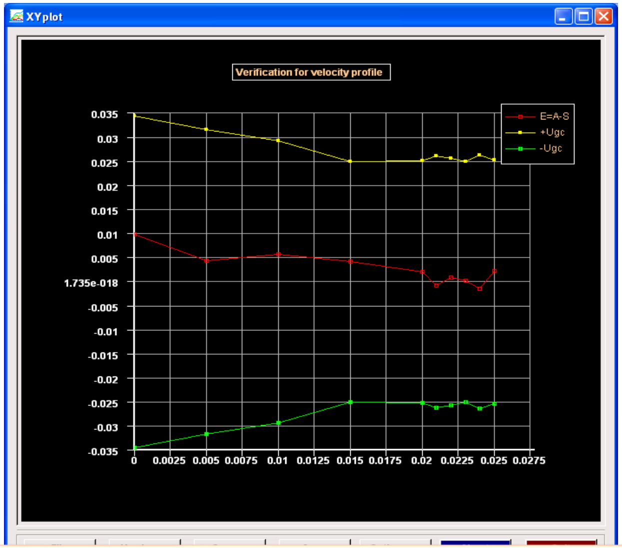
- Asymptotic Range: For sufficiently small $\Delta x_k$, the solutions are in the asymptotic range such that higher-order terms are negligible and the assumption that $p_k^{(i)}$ and $g_k^{(i)}$ are independent of $\Delta x_k$ is valid.
- When Asymptotic Range reached, $p_k$ will be close to the theoretical value $p_{k_{est}}$, and the correction factor $C_k$ will be close to 1.
- To achieve the asymptotic range for practical geometry and conditions is usually not possible and number of grids m>3 is undesirable from a resources point of view

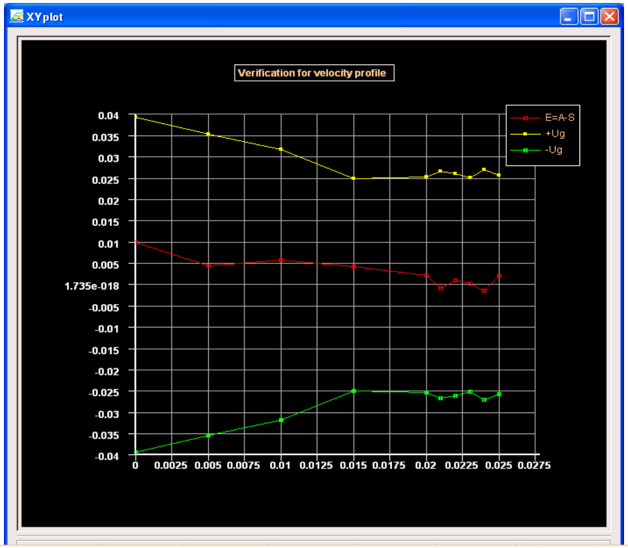
- **Verification for velocity profile using AFD:** To avoid ill-defined ratios, L2 norm of the $\varepsilon_{G21}$ and $\varepsilon_{G32}$ are used to define $R_G$ and $P_G$

$$\langle R_G \rangle = \left\| \varepsilon_{G_{21}} \right\|_2 \Big/ \left\| \varepsilon_{G_{32}} \right\|_2 \qquad \langle p_G \rangle = \frac{\ln\left( \left\| \varepsilon_{G_{32}} \right\|_2 \Big/ \left\| \varepsilon_{G_{21}} \right\|_2 \right)}{\ln(r_G)}$$

Where <> and $\| \ \|_2$ are used to denote a profile-averaged quantity (with ratio of solution changes based on L2 norms) and L2 norm, respectively.

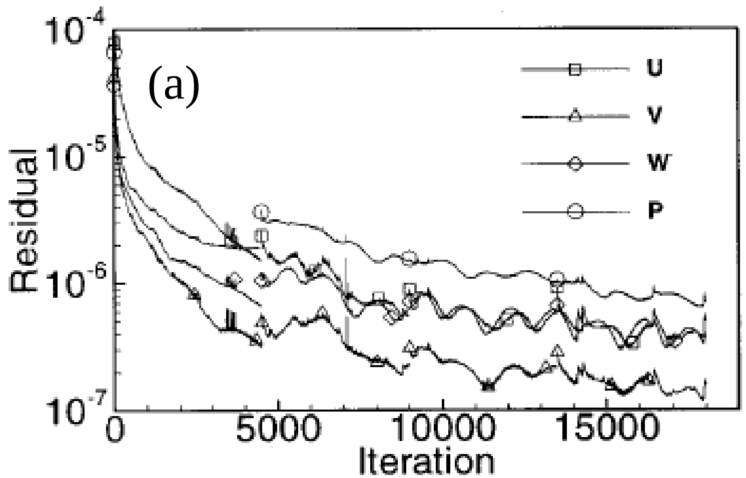
NOTE: For verification using AFD for axial velocity profile in laminar pipe flow (CFD Lab1), there is no modeling error, only grid errors. So, the difference between CFD and AFD, E, can be plot with +Ug and –Ug, and +Ugc and –Ugc to see if solution was verified.



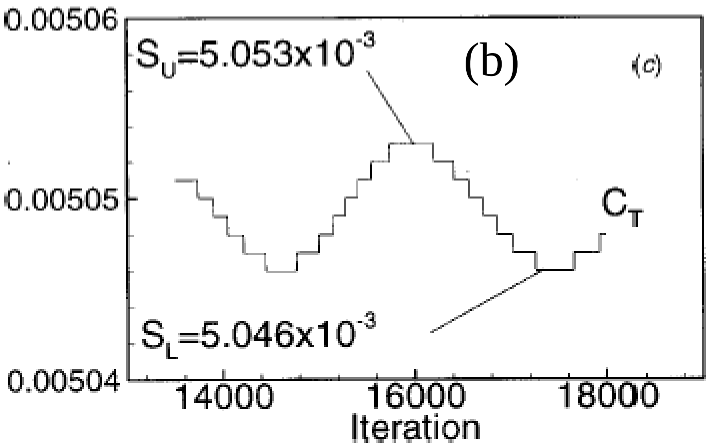



## Post-processing (Verification: Iterative Convergence)

- Typical CFD solution techniques for obtaining steady state solutions involve beginning with an initial guess and performing time marching or iteration until a steady state solution is achieved.
- The number of order magnitude drop and final level of solution residual can be used to determine stopping criteria for iterative solution techniques (1) Oscillatory (2) Convergent (3) Mixed oscillatory/convergent




$\frac{1}{2}(S_U - S_L)$




Iteration history for series 60: (a). Solution change (b) magnified view of total resistance over last two periods of oscillation (Oscillatory iterative convergence)

Post-processing (UA, Validation)

- Validation procedure: simulation modeling uncertainties was presented where for successful validation, the comparison error, E, is less than the validation uncertainty, Uv.
- Interpretation of the results of a validation effort

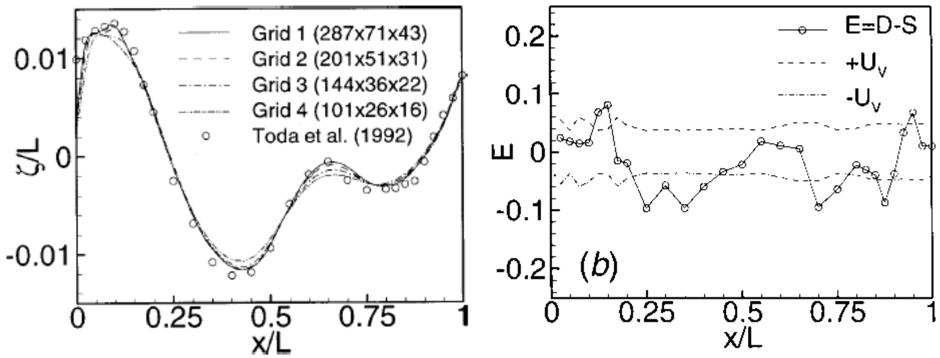
$|E| < U_V$ Validation achieved

$U_V < |E|$ Validation not achieved

$$E = D - S = \delta_D - (\delta_{SM} + \delta_{SN})$$

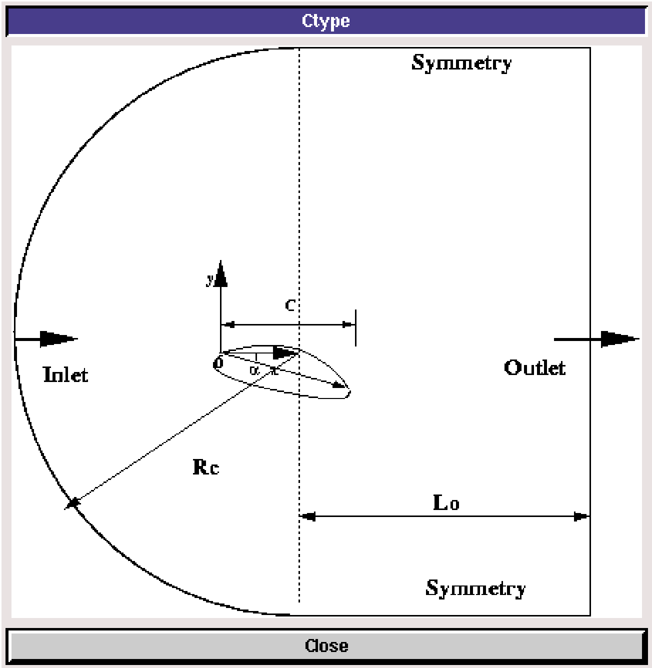
$$U_V = \sqrt{U_{SN}^2 + U_D^2}$$

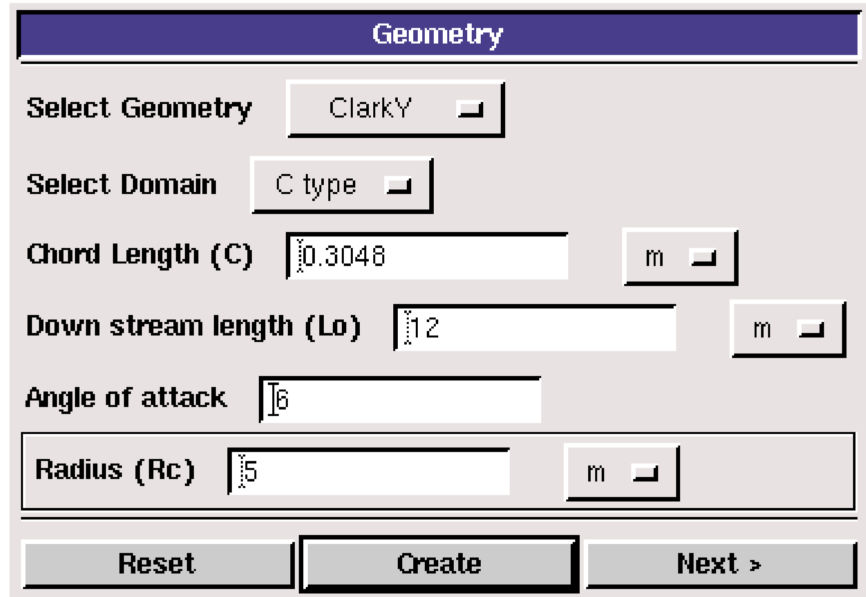
- Validation example




Example: Grid study and validation of wave profile for series 60

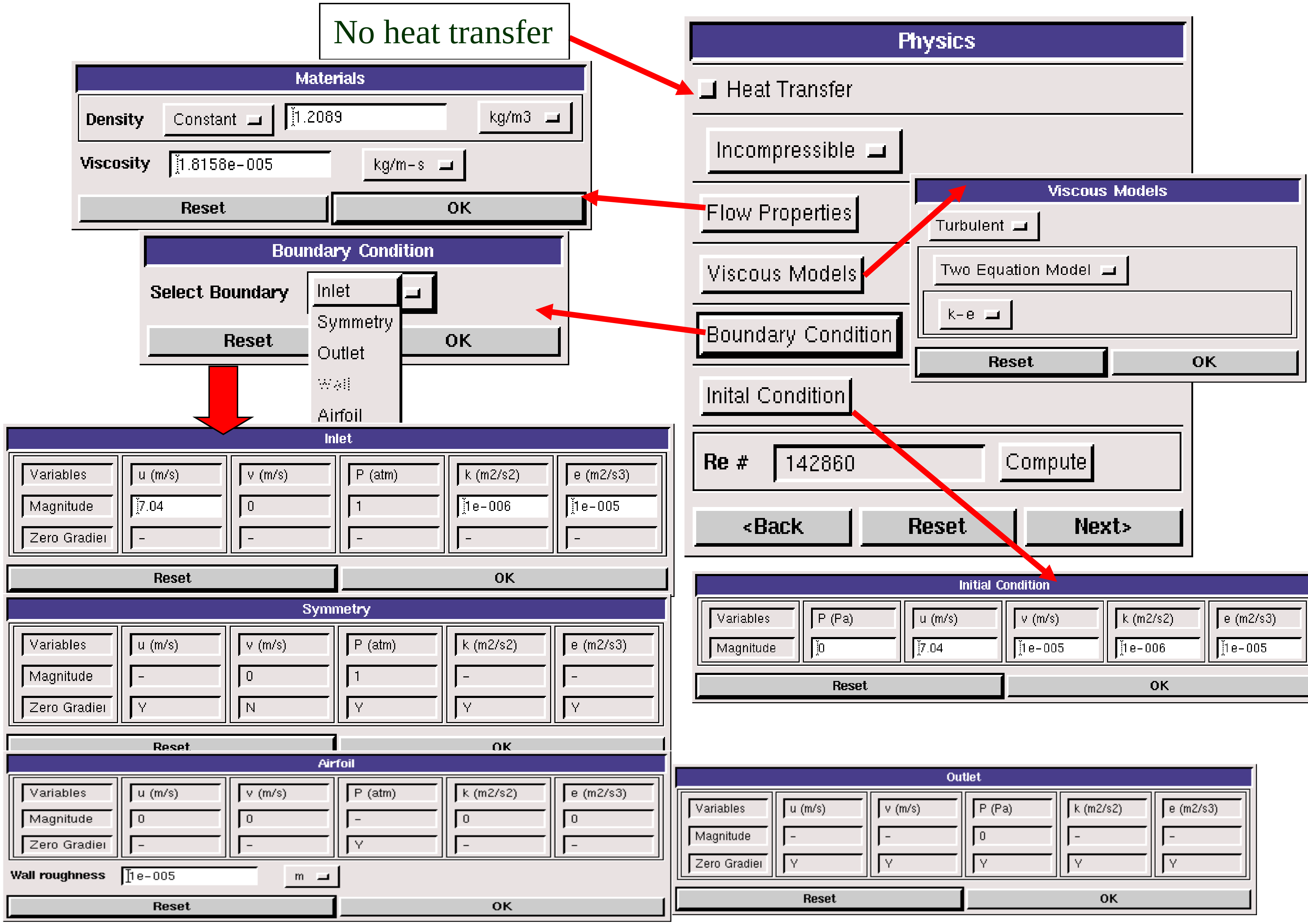
## Example of CFD Process using CFD educational interface (Geometry)

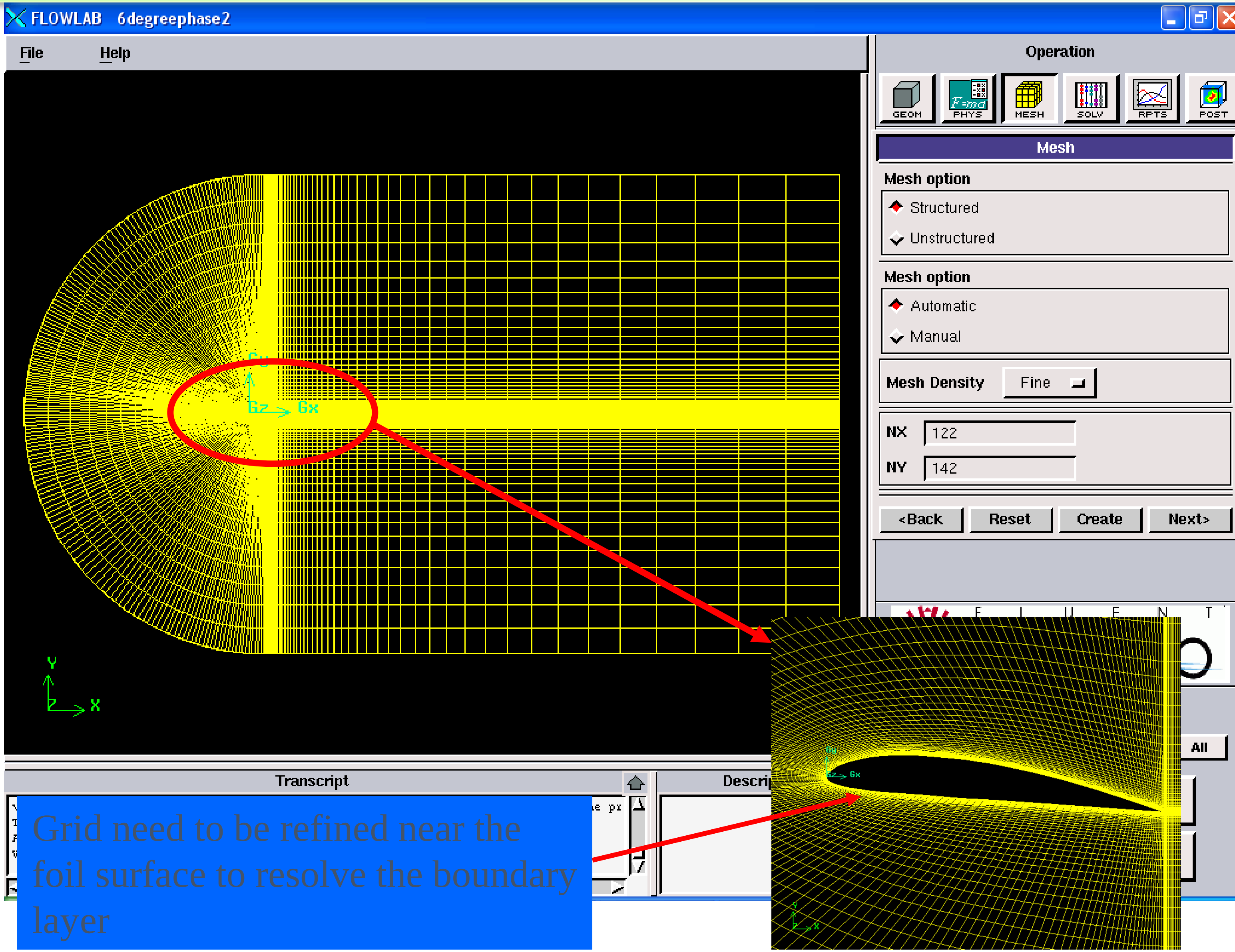


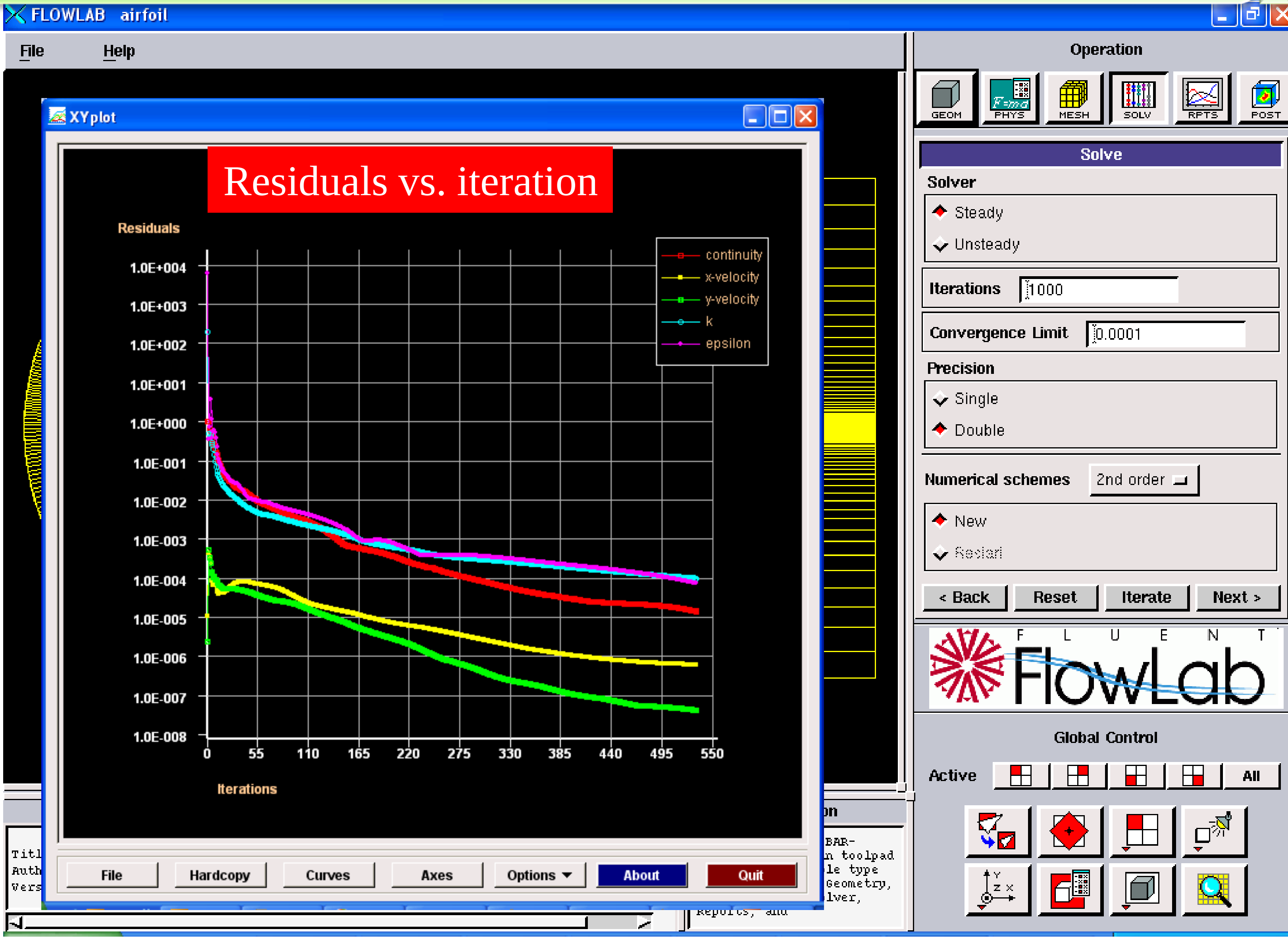

- Turbulent flows (Re=143K) around Clarky airfoil with angle of attack 6 degree is simulated.
- “C” shape domain is applied
- The radius of the domain Rc and downstream length Lo should be specified in such a way that the domain size will not affect the simulation results

# Example of CFD Process (Physics)

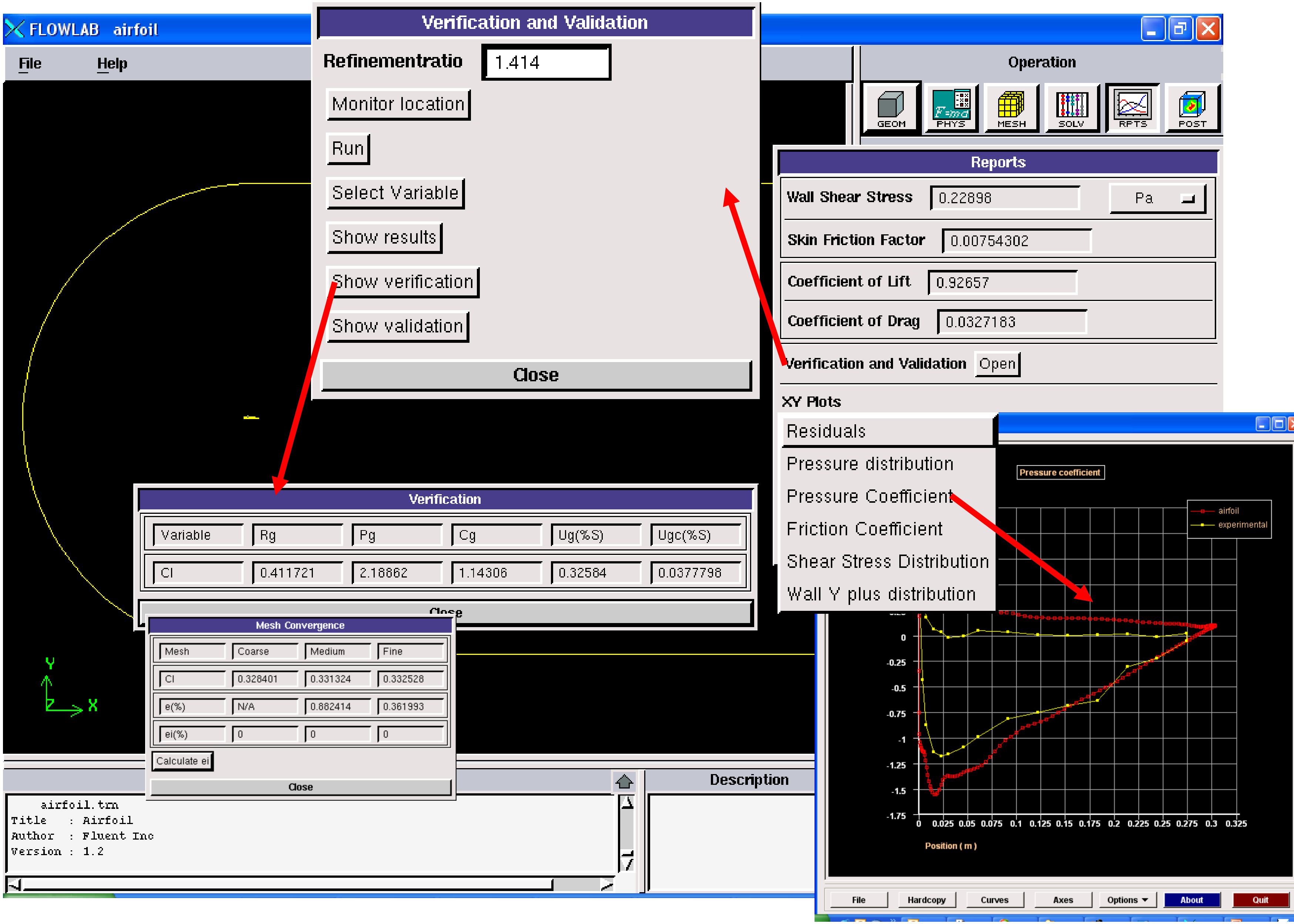




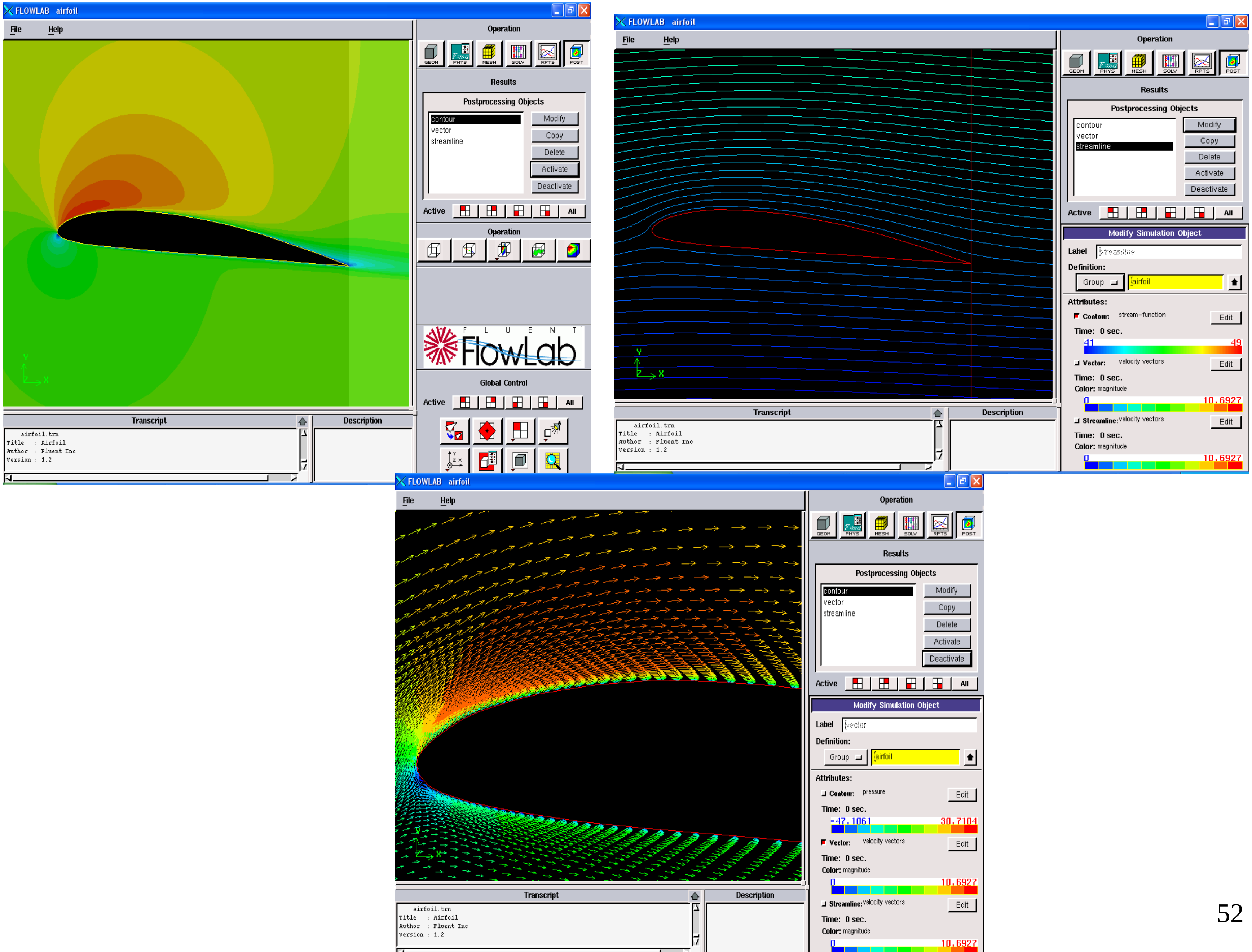

Grid need to be refined near the foil surface to resolve the boundary layer


FLOWLAB airfoil
File
Help
XYplot
Residuals vs. iteration
Residuals
continuity
x-velocity
y-velocity
k
epsilon
1.0E+004
1.0E+003
1.0E+002
1.0E+001
1.0E+000
1.0E-001
1.0E-002
1.0E-003
1.0E-004
1.0E-005
1.0E-006
1.0E-007
1.0E-008
0 55 110 165 220 275 330 385 440 495 550
Iterations
File Hardcopy Curves Axes Options About Quit
Operation
GEOM PHYS MESH SOLV RPTS POST
Solve
Solver
Steady
Unsteady
Iterations 1000
Convergence Limit 0.0001
Precision
Single
Double
Numerical schemes 2nd order
New
Restart
< Back Reset Iterate Next >
FLUENT
FlowLab
Global Control
Active
All

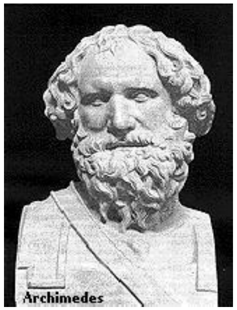
# Example of CFD Process (Reports)



# Example of CFD Process (Post-processing)



# Faces of Fluid Mechanics




Archimedes
(C. 287-212 BC)

Newton
(1642-1727)


Leibniz
(1646-1716)

Bernoulli
(1667-1748)

Euler
(1707-1783)

Navier
(1785-1836)

Stokes
(1819-1903)

Reynolds
(1842-1912)

Prandtl
(1875-1953)

Taylor
(1886-1975)

Kolmogorov
(1903-1987)